

# Motorway Planning.

## The Motorway Planning problem.



Making roads is expensive.

Making motorways is even more expensive.

Four towns exist at the corners of a huge square. The sides are 10 km in length.

The Government decides to build a motorway to connect each of towns.

They need to keep the costs down and so keep the total length of the motorway to a minimum.

Which of these designs do you think the planners should use?

### Task One.

What is the total length of the motorway?

Comment: Could be done but is there a cheaper plan?

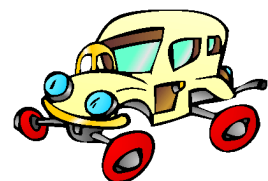
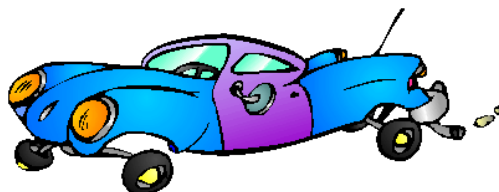
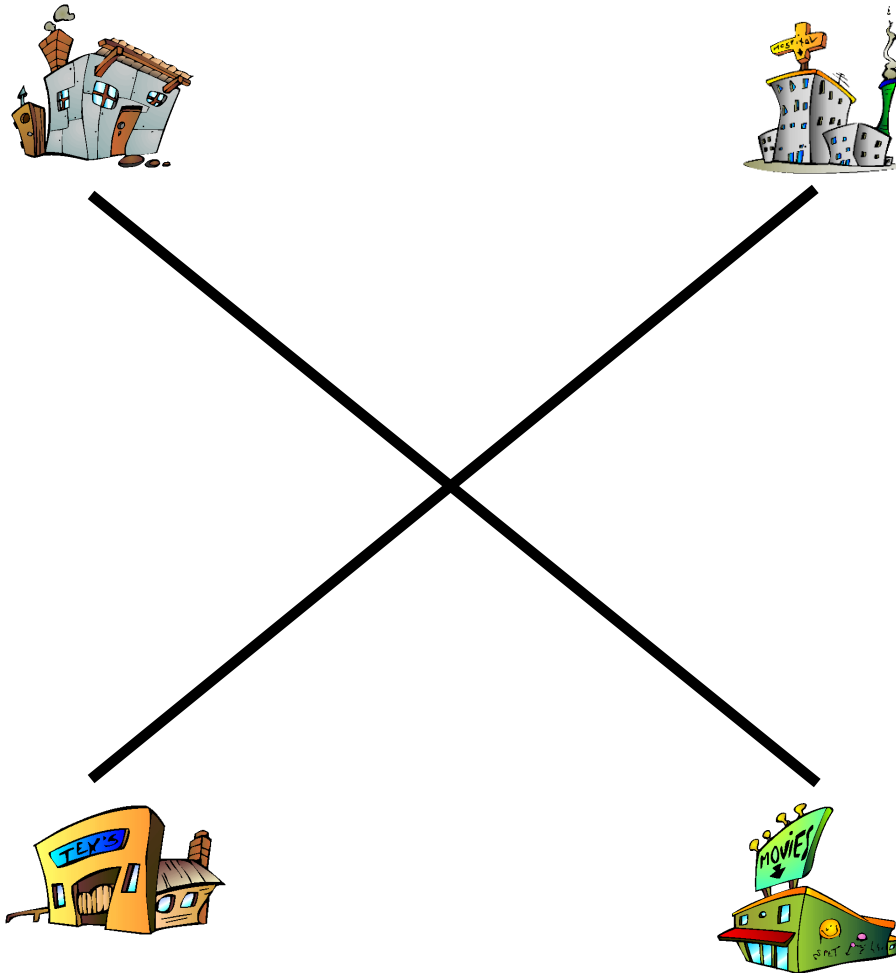


## Task Two.

What is the total length of the motorway?

Suggestions: Use Pythagoras Theorem to calculate the length of the lines.  
Or draw the diagram to scale and measure the lines.

Comment: Not good. There are no direct links between towns?



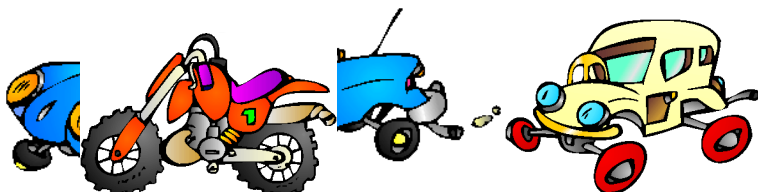
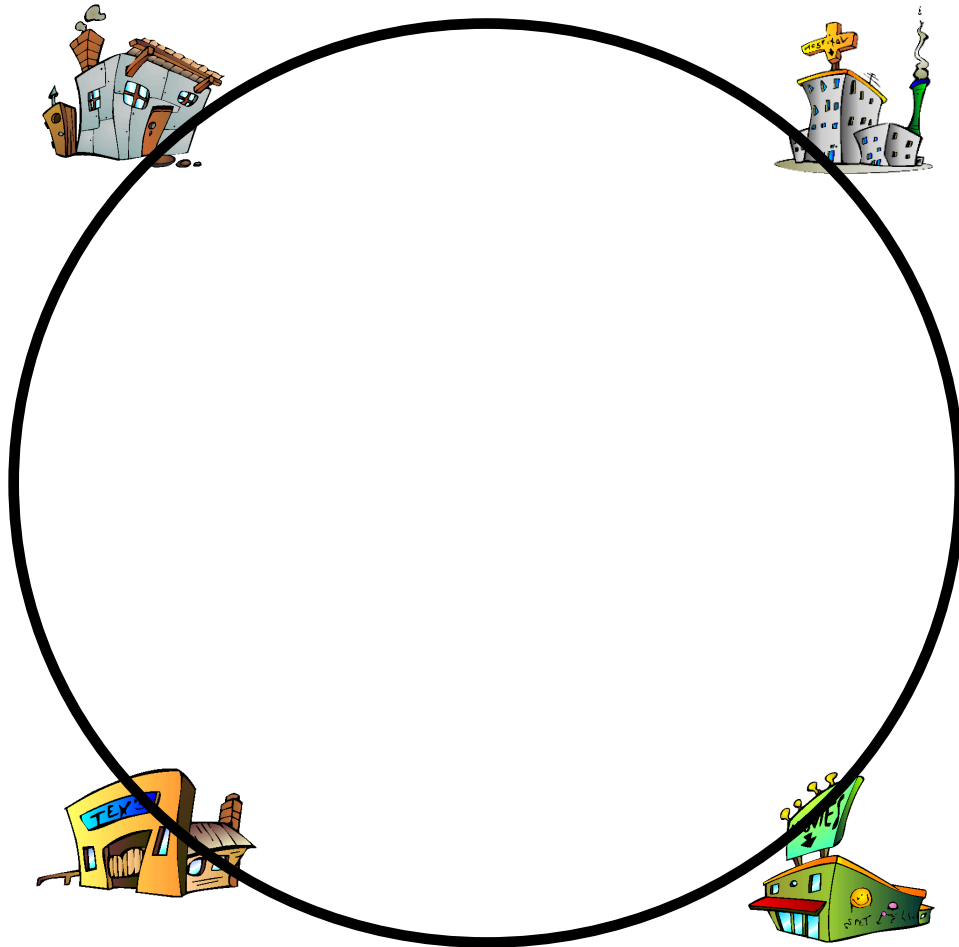
## Task Three.

What is the total length of the motorway?

Suggestions: The diameter of the circle can be calculated using Pythagoras.  
The circle could be drawn and then the diameter measured.

The circumference formula is  $C = \pi \times d$

Comment: Not comfortable driving in a circle all the time!

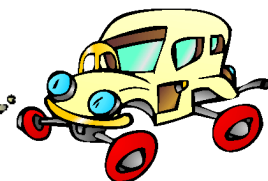
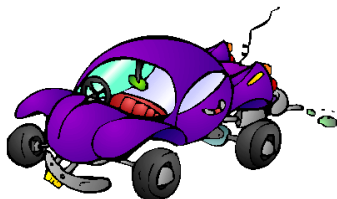
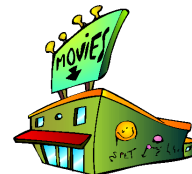


## Task Four.

What is the total length of the motorway?

Suggestions: Just measure or calculate the length of the lines.

Comment: Not good. There is no direct link between the two top towns.  
Is this the shortest distance so far?

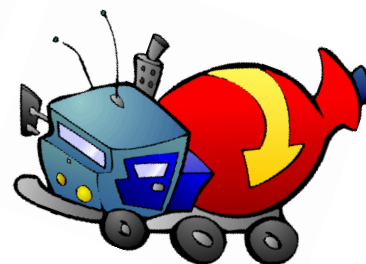
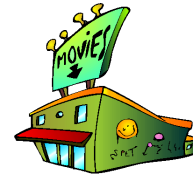
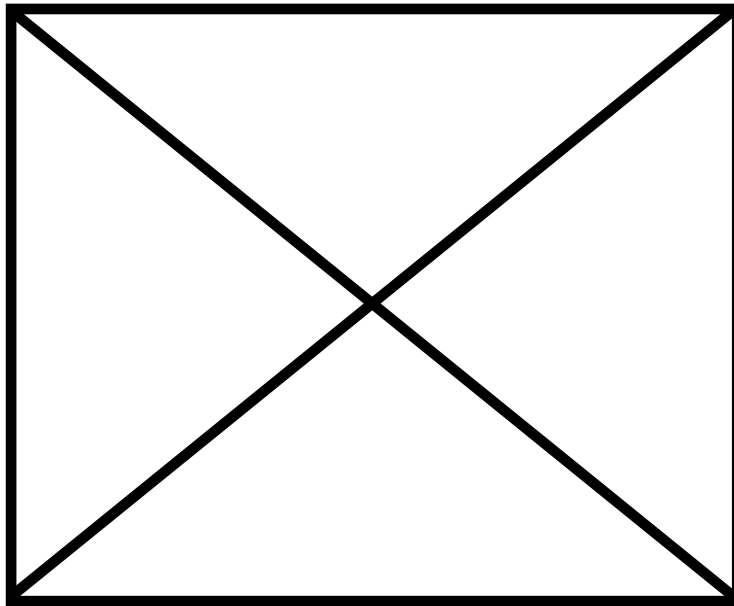


## Task Five.

What is the total length of the motorway?

Suggestions: Use Pythagoras Theorem to calculate the length of the lines.  
Or draw the diagram to scale and measure the lines.

Comment: This certainly links all the towns.  
Could be done but is there a cheaper plan?

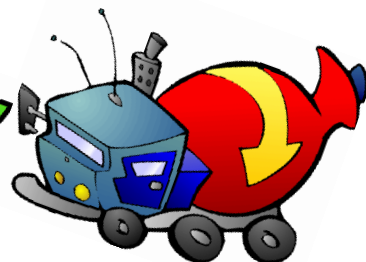
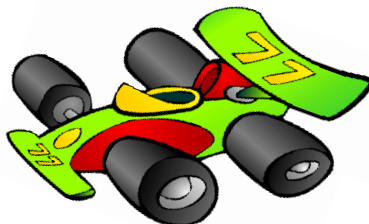
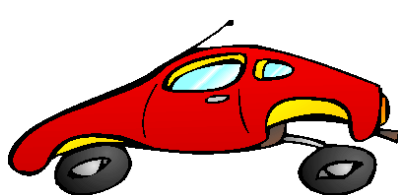
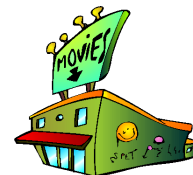
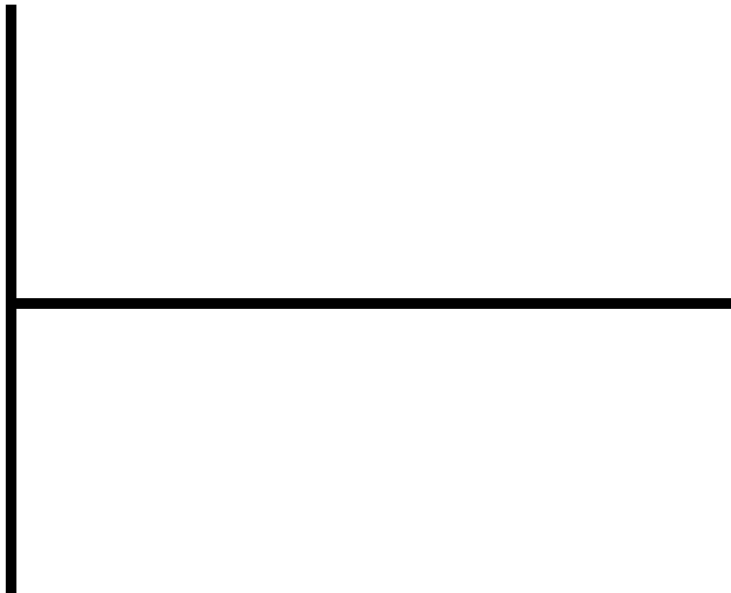


## Task Six.

What is the total length of the motorway?

Suggestions: Calculate the distance using the original square.  
Or draw the diagram to scale and measure the lines.

Comment: Joins all the towns rather well.  
Could be done but is there a cheaper plan?

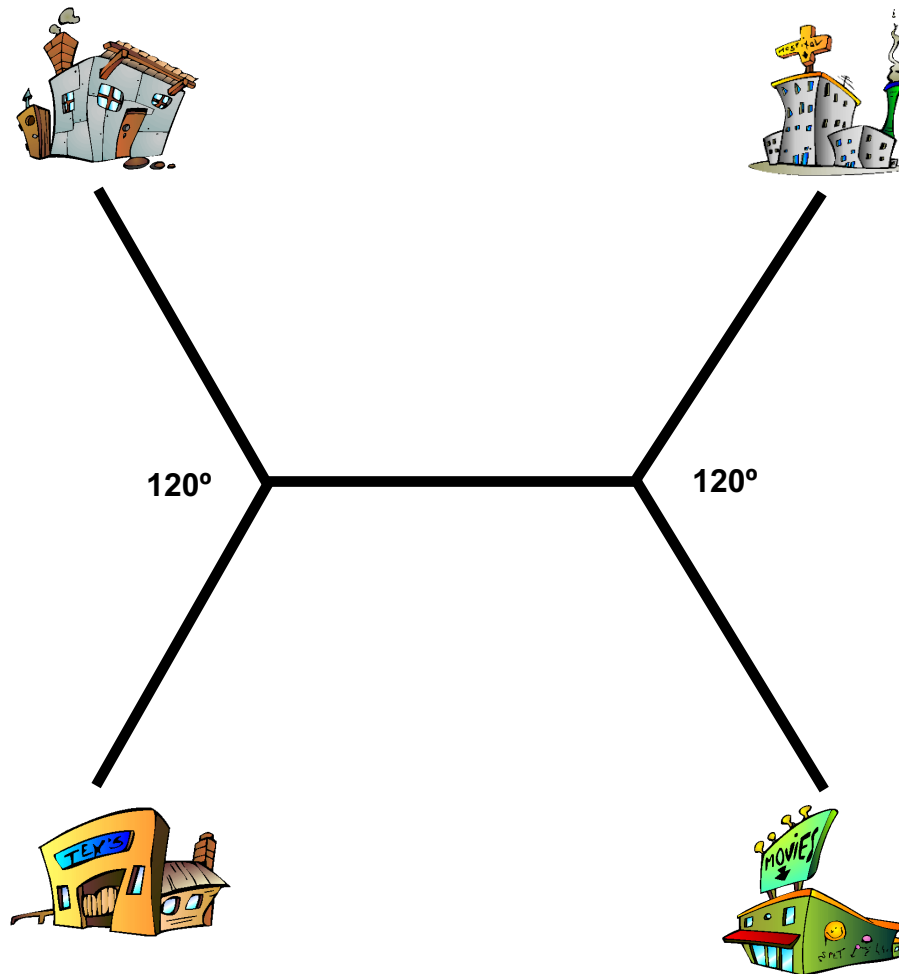


## Task Seven.

What is the total length of the motorway?

**Suggestions:** The angle measure of  $120^\circ$  is important.  
Use Trigonometry to calculate the length of the lines.  
Or draw the diagram to scale and measure the lines.

**Comment:** This joins all the towns rather well.  
Is it the cheaper plan?  
If it is. How much cheaper is it?



## Task Eight.

It is interesting that the solutions to these motorway tasks can be found using an experiment with soapy water!!  
This is explained and shown at

<https://www.youtube.com/watch?v=dAyDi1aa40E>

Have a look at this video!





## Teacher's notes and answers.

### Task One.

Answer by calculation is 40.

Answer by scale drawing and measuring is 40.

### Task Two.

Answer by calculation is  $20\sqrt{2}$  or 28.2

Answer by scale drawing and measuring is 28

### Task Three.

Answer by calculation is  $20\pi\sqrt{2}$  or 88.9

Answer by scale drawing and measuring is 89

### Task Four.

Answer by calculation is 30

Answer by scale drawing and measuring is 30

### Task Five.

Answer by calculation is  $40 + 20\sqrt{2}$  or 68.3

Answer by scale drawing and measuring is 68

### Task Six.

Answer by calculation is 30

Answer by scale drawing and measuring is 30

### Task Seven.

Answer by calculation is  $10 + 10\sqrt{3}$  or 27.3

Answer by scale drawing and measuring is 27

### Task Eight.

Video showing minimum length using soap mix.

<https://www.youtube.com/watch?v=dAyDi1aa40E>

### Notes for Teachers.

- This is an unusual topic and leads on to Steiner Points. They are described in the YouTube video if students want to go further.
- The younger students or students who haven't experienced the wonders of trigonometry can do these tasks by construction and measuring.
- They are very good tasks involving the Pythagoras Theorem and Trigonometry.
- All the tasks are based around a square with length 10. By scale this makes for easy constructions and measuring.
- The YouTube video is excellent and if unable to play it on the school network students can play it at their homes on the family's internet.
- Extension could be 3 Towns or 5 Towns. This introduces Steiner Points and Trees.